



## Idealized P-Matrix Based Modelling and Computational Analysis of SAW Delay Lines for Improved Performance in Sensors

T. Venkatesan<sup>1</sup>, R. Banu Priya<sup>2</sup>, G. Pandiyarajan<sup>3</sup>, Haresh M. Pandya<sup>4\*</sup>

<sup>1,3,4\*</sup> Department of Physics, Chikkanna Government Arts College, Tiruppur, TN, India.

<sup>2</sup> Department of Physics, Gobi Arts & Science College, Gobichettipalayam, TN, India.

Received: 25.10.2015 Accepted: 30.11.2015

### Abstract

The present research study describes the computational P-matrix model based modelling and subsequent analysis of the frequency response of a 300 MHz ST-X Quartz Surface Acoustic Wave (SAW) delay line device designed and fabricated with 43.5 IDT finger pairs. Employing a custom made MATLAB® algorithm, the device is modelled with a provision to vary its design parameters. Finally, a comparison of the computationally simulated frequency response with experimental device results indicates promising and good agreement between model and experiment which can be utilized and implemented during design and development stage of these devices for their potential applications in SAW Sensor systems.

**Keywords:** Interdigital Transducer (IDT); MATLAB®; P-Matrix Model; SAW Delay Lines; SAW Sensors.

## 1. INTRODUCTION

Surface Acoustic Wave (SAW) sensors (Raj *et al.* 2013) are extremely versatile devices that are commercially possible for transduction of physical and chemical quantities. Almost all physical quantities like Surface mass (Hribšek *et al.* 2010), Stress (Filipiak *et al.* 2007), Strain (Hempel *et al.* 2013), temperature (Malocha *et al.* 2013), Pressure (Benetti *et al.* 2008) etc., can be measured by SAW sensors. SAW sensors are miniaturized, highly sensitive, reproducible, long term stable and fast real time response (Banu Priya *et al.* 2014).

Computer models for SAW devices are continuously improved to meet the demanding design requirements for key components in sensor applications (Amani *et al.* 2012; Chu *et al.* 2013; Sharma *et al.* 2014). This often leads to the increase in simplicity of the simulation codes and reduce in run time of the program. Recently, some sophisticated models have been developed. In our present study, a straight forward approach like P-Matrix model is employed (Rusakov *et al.* 2001). It provides fast and accurate simulation, which are used for designing SAW devices (Pastureau, 2004).

Modelling of SAW devices is resorted to achieve two aims :

(i) to understand propagation, generation and detection of surface waves in piezoelectric materials and

(ii) to study and design the structures such as IDTs, delay lines, filters, resonators, sensors etc., to achieve preferred device responses.

Modelling techniques need to provide fast and exact results, be comfortable to compute and distinctly offer relation between modelled parameters and device performance. A ST-X Quartz based 300 MHz SAW delay line with uniform IDTs possessing 43.5 finger pairs per IDT has been generated by using a MATLAB® software algorithm. Modelled device parameters are assumed, studied and graphically represented. The whole purpose of this simulation study is to model the SAW delay line for enhanced device performance and subsequently compare the results obtained from the experimental one (Haresh *et al.* 2010).

## 2. THEORETICAL ANALYSIS

### 2.1 P-Matrix modelling

The P-Matrix has been invented by Tobolka in 1979 (Manner *et al.* 1996) and is now widely used (Ruppel *et al.* 1994; Brown *et al.* 1989; Ventura *et al.* 1992) (Hartmann and Abbott, 1988). It is a mixed

\*Haresh M. Pandya Tel. no: +91 9894336750

Email: haresh.pandya@rediffmail.com

matrix representation of an IDT with three ports (Fig. 1). This three port has one electrical and two acoustical ports. To describe a whole transducer, the standard cell (Smith *et al.* 1972) three ports have to be cascaded acoustically whereas the electrical ports must be connected in parallel. The standard cell originally proposed by Smith *et al.* 1972 (Tancrrell *et al.* 1969; Tancrrell and Holland, 1971) represents one single finger and is described by means of the crossed-field equivalent circuit. Therefore, transducers are represented here by a mixed matrix, so called P-matrix, defined in this correspondence. The voltage-to-SAW transfer functions, the electrical admittance and the SAW scattering matrix of the shorted transducer are elements of the P-matrix, which will be calculated for a standard cell and the whole transducer. The two acoustic ports are described by an S-Matrix, and the electric port is described by an admittance.

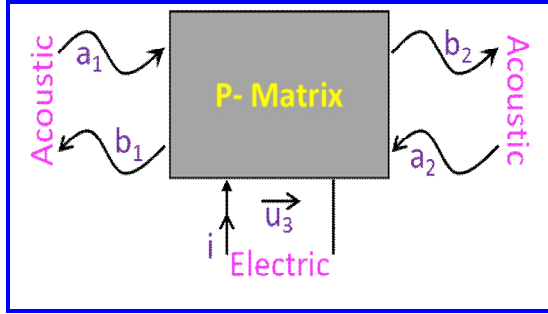


Fig. 1: Diagram showing mixed matrix variables at each port

P-matrix model is a distinct form of COM model, a well-established tool to consider the electrical and acoustical properties of IDT's and reflector gratings (Tobolka, 1979). This matrix is convenient for analysing a wide variety of SAW devices. This model relates both outgoing acoustic surface waves  $A_{i1}/A_{i2}$  and the electric current  $I$  to both incoming acoustic surface waves  $A_{i1}/A_{i2}$  and the electric voltage  $V_t$  (shown in Fig. 2).

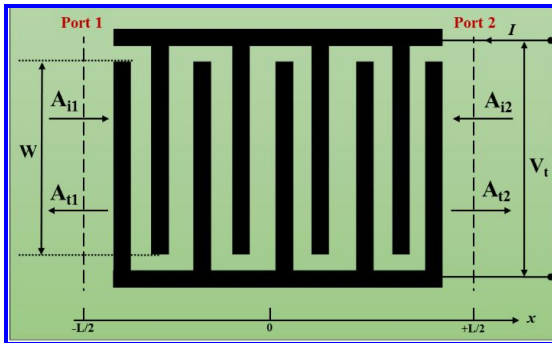


Fig. 2: P-Matrix Parameters of transducer

The P-matrix is defined such that,

$$\begin{bmatrix} A_{t1} \\ A_{t2} \\ I \end{bmatrix} = \underbrace{\begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix}}_{\text{P - matrix}} \begin{bmatrix} A_{i1} \\ A_{i2} \\ V_t \end{bmatrix} \quad (1)$$

Here,  $P_{33}$  denotes the transducer admittance  $Y_t$  (scalar),  $P_{11}$ ,  $P_{12}$ ,  $P_{21}$  and  $P_{22}$  are scattering parameters (matrices),  $P_{13}$  and  $P_{23}$  are electro-acoustic excitation matrix (vectors) and  $P_{31}$  and  $P_{32}$  are acousto-electric detection matrix (vectors). The value of  $P_{12}$  and  $P_{21}$  elements are non-zero from the basic cell idea introduced by (Ruppel *et al.* 1994). The most complicated one is the excitation matrix, where all  $P_{ij}$  elements are non-zero. The parameters of the above P-Matrix are given below

$$\begin{aligned} P_{11} &= P_{22} = 0 \\ P_{12} &= P_{21} = \exp(-jkL) \\ P_{13} &= -P_{31}/2 = j\bar{\rho}_e(k)\sqrt{\omega W \overline{s}/2} \exp(-jkL/2) \\ P_{23} &= -P_{32}/2 = j\bar{\rho}_e(-k)\sqrt{\omega W \overline{s}/2} \exp(-jkL/2) \\ P_{33} &= Y_t(\omega) = G_a(\omega) + jB_a(\omega) + j\omega C_t \end{aligned} \quad (2)$$

From the above equations, we define a function  $\rho_e(x)$  as the electrostatic charge density on the electrodes when unit voltage is applied and is calculated by a Green's function method (Baghai-Wadji *et al.* 1984).  $k$  is the wave number for waves between the two port location,  $L$  is the length of the IDT. The constant  $\overline{s}$  is defined as

$$\overline{s} = \frac{(\Delta v / v)}{\epsilon_\infty} \quad (2)$$

From the above equation  $\epsilon_\infty$  is known as effective permittivity (Guerra-Pulido and Pérez-Alcázar, 2014). The conductance  $G_a(\omega)$  can be written as

$$G_a(\omega) = \omega W \overline{s} |\overline{\rho}_e(k)|^2 \quad (3)$$

Where,  $\omega$  is angular frequency and  $W$  is the aperture of uniform IDT. The susceptance  $B_a(\omega)$  can be calculated from the equation,

$$B_a(\omega) = \frac{\omega W}{\pi} \left[ \frac{1}{k_f} \left| \overline{\rho_e(k_f)} \right|^2 \right] \quad (5)$$

The insertion loss for the SAW device can be determined by the quantity  $-20 \log I_p$  in dB, is

$$I_p = P_L / P_s = \frac{2G_a G_L}{|P_{33} + Y_L|^2} \quad (6)$$

Where  $G_L = \text{Re}\{Y_L\}$ .  $P_s$  and  $P_L$  are source power and power deliver to the load.

### 2.1 Cascading of two port components

In the present study, we considered two adjacent transducers with uniform aperture  $W$  and P-matrices  $P_{ij}^a$  and  $P_{ij}^b$  for SAW devices as shown in Fig. 3.

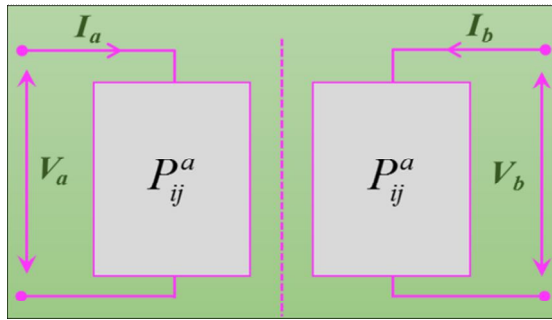


Fig. 3: P-Matrix for two component device

For a symmetric device structure, the admittance matrix elements (Y-parameters) of the two port SAW delay line can be written as

$$Y_{11} = P_{33}^a - \frac{2P_{11}^b (P_{13}^a)^2 e^{-jkl_d}}{1 - (P_{11}^a P_{11}^b) e^{-jkl_d}} \quad (7)$$

$$Y_{12} = Y_{21} = \frac{-2(P_{13}^a P_{13}^b) e^{jkl_d}}{1 - (P_{11}^a P_{11}^b) e^{-jkl_d}} \quad (8)$$

Where  $Y_{11}$  and  $Y_{21}$  represent the input and output (transfer) admittance (Guerra-Pulido and Pérez-Alcázar, 2014). Using the admittance matrices, the transfer function of the device can be obtained. For a two-port resonator device is connected to an arbitrary source and load, then by using Y to S conversion relations, the device transfer function can be written as

$$S_{12} = S_{21} = \frac{2Y_{12} \sqrt{R_1 R_2}}{Y_{12}^2 R_1 R_2 - (1 + Y_{11} R_1)(1 + Y_{22} R_2)} \quad (9)$$

The above equation gives the complete response of the SAW devices that has been derived by P-matrix.

### 2.2 Modelling strategy

The various steps adapted for modelling of SAW devices are highlighted as follows:

1. In present study ST-X Quartz substrate was selected. Because compare to other substrates, quartz has a good temperature stability, the temperature coefficient of delay (TCD) is nearly zero (Campbell, 1989).
2. IDT material chosen is Aluminium (Al). Because Al serves as an inert metal as well as good adhesive (Venkatesan and Pandya, 2013).
3. The input parameters for SAW device modelling according to P-matrix model is shown in Table 1.

Table 1. Input parameters for modelling of SAW Device

S. No	Parameter (symbol)	Values
1	Coupling Coefficient ( $k^2$ )	0.0016 (ST-X Quartz)
2	SAW Velocity ( $V_s$ )	3158 m/s
3	Operating frequency ( $f_0$ )	300 MHz
4	IDT geometry	Split geometry
5	Number of finger pairs ( $N_p$ )	43.5 finger pairs
6	Load and Source resistance ( $R_L$ and $R_s$ )	50 $\Omega$

4. Employing the P-Matrix model, the outputs are shown in graphical format to compare experimental results.
5. An operating center frequency of 300 MHz was modelled using a MATLAB algorithm and simulated results of SAW device output graphs were obtained. The modelling strategy adapted as showing Fig. 7.

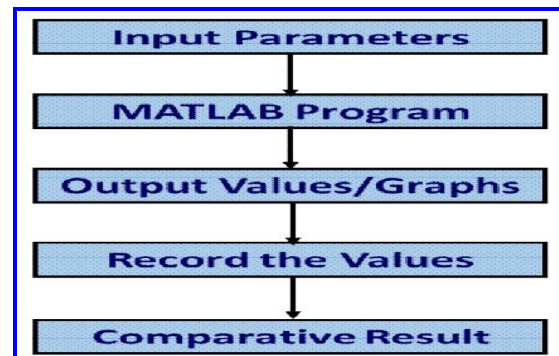


Fig. 7: Flow chart of Modelling Strategy

### 3. RESULT & DISCUSSION

As previously declared, an ST-X Quartz piezoelectric substrate based 300MHz SAW delay line with 43.5 finger pairs, split electrode geometry and constant aperture was fabricated in-house employing laser pattern writing for mask making, thermal evaporation for metallization and Photolithographic (PLG) etching method for device making. This was followed by the standard procedures of dicing, bonding and packaging in the form of a portable unit (Haresh M. Pandya *et al.* 2013).

The modelled device response was calculated as given by equation (9). The steps involved in the modelling procedure were codified into an intensely computative MATLAB® program which was run many times by varying the free surface SAW velocity ( $v_0$ ) for ST-X Quartz piezoelectric substrate around 3158 m/s. The results are also exported to .xls format for comparison with the experimental values obtained. The MATLAB® algorithm was codified to further generate plots of modelled parameters as a function of frequency so as to deduce their behavior directly.

In addition to the above, the algorithm also had the flexibility to simulate device parameters for various sets of SAW delay lines for the device designer. The striking point of this modelling exercise was that the specific frequency response was obtained for the 300 MHz SAW delay line. The modelled parameters received from the P-Matrix Model are summarized in Table 2.

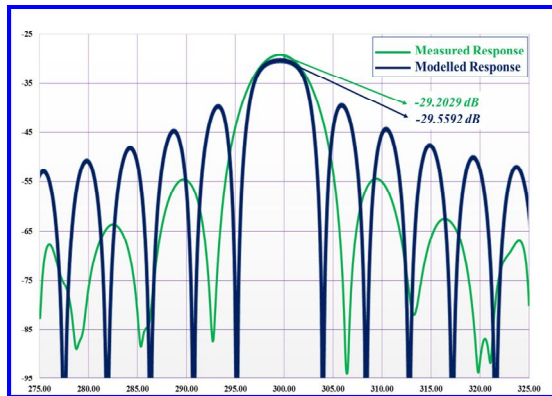


Fig. 8. Comparison of Modelled and Measured Values

A comparison of the modelled frequency response with the measured values (Fig. 8) demonstrates in an excellent manner, the near perfect match achieved in the frequency response plots which account for second order effects such as EMF, TTE and circuit parasitics. All important parameters like

observed frequency response, insertion loss and 3dB bandwidth are accurately modelled and predicted. A difference of around 0.3583 dB is observed between the modelled value of insertion loss -29.5592 dB and its measured value of -29.2029 dB, the difference of around 0.3660 MHz is observed between the modelled value of 3dB bandwidth 4.1315 MHz and its measured value of 4.4975 MHz and frequency responses also nearly same.

It can be clearly seen that there is a slight difference in the modelled values of IL as shown in

Table 2. Comparison of Modelled and Measured Values

S. No	Parameter (symbol)	Modelled Values	Measured Values
1	Operating frequency ( $f_0$ )	299.99 MHz	299.59 MHz
2	Insertion Loss	-29.5592 dB	-29.2029 dB
3	3dB Bandwidth	4.1315 MHz	4.4975 MHz

### 4. CONCLUSION

The present research study describes the computational P-matrix model based modelling and subsequent analysis of the frequency response of a 300 MHz ST-X Quartz Surface Acoustic Wave (SAW) delay line device designed and fabricated with 43.5 IDT finger pairs.

Employing a custom made MATLAB® algorithm, the device is modelled with a provision to vary its design parameters. Finally, a comparison of the computationally simulated frequency response with experimental device results indicates promising and good agreement between model and experiment which can be utilized and implemented during design and development stage of these devices for their potential applications in SAW Sensor systems.

### REFERENCE

- Amani, M., Chu, Y., Waterman, K. L., Hurley, C. M., Platek, M. J. and Gregory, O. J., Detection of triacetone triperoxide (TATP) using a thermodynamic based gas sensor, *Sens. Actuat. B Chem.*, 162, 7-13(2012).  
doi:10.1016/j.snb.2011.11.019
- Baghai-Wadji, A. R., Selberherr, S. and Seifert, F., On the calculation of charge, electrostatic potential and capacitance in generalized finite saw structures, In: IEEE 1984 Ultrasonics Symposium, *IEEE*, 44-48(1984).  
doi:10.1109/ULTSYM.1984.198261

- Banu Priya, R., Venkatesan, T., Pandiyarajan, G. and Haresh M. Pandya, SAW devices - A comprehensive review, *J. Environ. Nanotechnol.*, 3, 106-115(2014).  
[doi:10.13074/jent.2014.09.143101](https://doi.org/10.13074/jent.2014.09.143101)
- Baskar Raj, V., Singh, H., Nimal, A. T., Tomar, M., Sharma, M. U. and Gupta, V., Effect of metal oxide sensing layers on the distinct detection of ammonia using surface acoustic wave(SAW) sensors, *Sens. Actuat. B Chem.*, 187, 563-573(2013).  
[doi:10.1016/j.snb.2013.04.063](https://doi.org/10.1016/j.snb.2013.04.063)
- Benetti, M., Cannata, D., Di Pietrantonio, F., Marchiori, C., Persichetti, P. and Verona, E., Pressure sensor based on surface acoustic wave resonators, In: 2008 IEEE Sensors, *IEEE*, 1024-1027(2008).  
[doi:10.1109/ICSENS.2008.4716617](https://doi.org/10.1109/ICSENS.2008.4716617)
- Brown, R. B., Horine, B. H. and Hines, J. H., Practical implementation of coupling-of-modes theory for SAW device modeling, In: Proceedings., IEEE Ultrasonics Symposium, *IEEE*, 153-158(1989).  
[doi:10.1109/ULTSYM.1989.66975](https://doi.org/10.1109/ULTSYM.1989.66975)
- Campbell, C. K., Applications of surface acoustic and shallow bulk acoustic wave devices, Proceedings of the IEEE, *IEEE*, 77, 1453-1484(1989).  
[doi:10.1109/5.40664](https://doi.org/10.1109/5.40664)
- Chu, Y., Mallin, D., Amani, M. and Gregory, O. J., Detection of explosives using orthogonal gas sensors, In: 2013 IEEE Sensors, *IEEE*, 1-4(2013).  
[doi:10.1109/ICSENS.2013.6688125](https://doi.org/10.1109/ICSENS.2013.6688125)
- Filipiak, J., Solarz, L. and Steczko, G., Surface acoustic wave stress sensors -Designing, *Mol. Quant. Acoust.*, 28, 71-80(2007).
- Guerra-Pulido, J. O. and Pérez-Alcázar, P. R., Generalised relations between P- and Y-matrices and their applications, *J. Eng.*, 1-8(2014).  
[doi:10.1049/joe.2014.0215](https://doi.org/10.1049/joe.2014.0215)
- Haresh M. Pandya, Nimal, A. T., Sharma, M., Gopalakrishnan, P., Mittal, U. and Elango, P., Equivalent circuit MATLAB modelling of a surface acoustic wave( SAW ) delay line for sensor applications, IISM-2010, 1-6(2010).
- Haresh M. Pandya, Sharma, M. U., Nimal, A. T. and Rajesh, K. B., Impulse modelled response of a 300 mhz st-quartz SAW device for sensor specific applications, *J. Environ. Nanotechnol.*, 2, 15-21(2013).  
[doi:10.13074/jent.2013.02.nciset33](https://doi.org/10.13074/jent.2013.02.nciset33)
- Hartmann, C. S. and Abbott, B. P., A generalized impulse response model for SAW transducers including effects of electrode reflections, In: IEEE 1988 Ultrasonics Symposium Proceedings, *IEEE*, 29-34(1988).  
[doi:10.1109/ULTSYM.1988.49337](https://doi.org/10.1109/ULTSYM.1988.49337)
- Hempel, J., Wilde, J. and Reindl, L. M., Effects of residual stress on assembled saw strain sensors, *PIERS*, 286-290(2013).
- Hribšek, M. F., Tosić, D. V., Tasić, M., Filipović, Z. and Živković, Z., Design and realization of transversal surface acoustic wave RF filters, In: 5th European Conference on Circuits and Systems for Communications (ECCSC), *IEEE*, 82-85(2010).
- Malocha, D., Gallagher, M., Fisher, B., Humphries, J., Gallagher, D. and Kozlovski, N., A passive wireless Multi-Sensor SAW technology device and system perspectives, *Sens.*, 13, 5897-5922(2013).  
[doi:10.3390/s130505897](https://doi.org/10.3390/s130505897)
- Manner, O., Wagner, K. C. and Ruppel, C. C. W., Advanced numerical methods for the simulation of SAW devices, IEEE Ultrasonics Symposium Proceedings, *IEEE*, 123-130(1996).  
[doi:10.1109/ULTSYM.1996.583820](https://doi.org/10.1109/ULTSYM.1996.583820)
- Pastureaud, T., Evaluation of the P-matrix parameters frequency variation using periodic FEM/BEM analysis, Ultrasonics Symposium, *IEEE*, 1, 80-84(2004).  
[doi:10.1109/ULTSYM.2004.1417673](https://doi.org/10.1109/ULTSYM.2004.1417673)
- Ruppel, C. C. W., Ruile, W., Scholl, G. and Wagner, K. C., Review of models for low-loss filter design and applications, Proceedings of IEEE Ultrasonics Symposium, *IEEE*, 1, 313-324(1994).  
[doi:10.1109/ULTSYM.1994.401602](https://doi.org/10.1109/ULTSYM.1994.401602)
- Rusakov, A. N., Orlov, V. S., Chao, B. and Lee, V., Application of modified P-matrix model to the simulation of radio frequency LSAW filters, In: 2001 IEEE Ultrasonics Symposium, Proceedings, *IEEE*, 7-11(2001).  
[doi:10.1109/ULTSYM.2001.991567](https://doi.org/10.1109/ULTSYM.2001.991567)
- Sharma, M. U., Kumar, D., Koul, S. K., Venkatesan, T., Pandiyarajan, G., Nimal, A. T., Kumar, P. R. and Haresh M. Pandya, Modelling of SAW devices for gas sensing applications -A comparison, *J. Environ. Nanotechnol.*, 3, 63-66(2014).  
[doi:10.13074/jent.2014.12.144110](https://doi.org/10.13074/jent.2014.12.144110)
- Smith, W. R., Gerard, H. M. and Jones, W. R., Analysis and design of dispersive interdigital surface-wave transducers, *IEEE Trans. Microw. Theory Tech.*, 20, 458-471(1972).  
[doi:10.1109/TMTT.1972.1127786](https://doi.org/10.1109/TMTT.1972.1127786)
- Tancrrell, R. H. and Holland, M. G., Acoustic surface wave filters, Proceedings of the IEEE, *IEEE*, 59, 393-409(1971).  
[doi:10.1109/PROC.1971.8180](https://doi.org/10.1109/PROC.1971.8180)
- Tancrrell, R. H., Schulz, M. B., Barrett, H. H., Davis, L. and Holland, M. G., Dispersive delay lines using ultrasonic surface waves, Proceedings of the IEEE, *IEEE*, 57, 1211-1213(1969).  
[doi:10.1109/PROC.1969.7202](https://doi.org/10.1109/PROC.1969.7202)
- Tobolka, G., Mixed Matrix Representation of SAW transducers, IEEE Transactions on Sonics and Ultrasonics, *IEEE*, 26, 426-427(1979).  
[doi:10.1109/T-SU.1979.31128](https://doi.org/10.1109/T-SU.1979.31128)



- Venkatesan, T. and Haresh M. Pandya, Surface acoustic wave devices and sensors -A short review on design and modelling by impulse response, *J. Environ. Nanotechnol.*, 2, 81-89(2013).  
[doi:10.13074/jent.2013.09.132034](https://doi.org/10.13074/jent.2013.09.132034)
- Ventura, P., Solal, M., Dufilie, P., Desbois, J., Doisy, M. and Hode, J. M., Synthesis of SPUDT filters with simultaneous reflection and transduction optimization, In: IEEE 1992 Ultrasonics Symposium Proceedings, *IEEE*, 71-75(1992).  
[doi:10.1109/ULTSYM.1992.276061](https://doi.org/10.1109/ULTSYM.1992.276061)